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Modified Product Estimators for Estimation of Population Variance using Known Parameters of an Auxiliary Variable

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Abstract—*The present paper deals with some modified product estimators for estimation of population variance of the study variable when the Median, Skewness, Kurtosis and Quartiles of the auxiliary variable are known. The bias and the mean squared error of the proposed estimators are derived and are compared with that of the product estimator for estimation of population variance. As a result we have derived the conditions for which the proposed estimators perform better than the product estimator. Further the performances of the proposed estimators with that of the product estimator are assessed for three different natural populations. From the numerical study it is observed that the proposed estimator performs better than the traditional product type variance estimator.*

Keywords*: Bias, Mean Squared Error, Natural Populations, Simple Random Sampling.*

1. INTRODUCTION

It is common practice to use the auxiliary variable for improving the accuracy of the estimate of a parameter. When the information on an auxiliary variable X is known, a number of estimators such as ratio, product and linear regression estimators are available in the literature. When the correlation between the study variable and the auxiliary variable is negative, product method of estimation is quite effective. Consider a finite population $U=\{U_1,U_2,...,U_N\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i=1,2,3,...,N$ giving a vector $Y=\{Y_1,Y_2,...,Y_N\}$. The problem is to estimate the population mean $\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i$ on the basis of a random sample selected from the population U or its variance $S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^2$. When there is no additional information on the auxiliary variable available, the simplest estimator of population mean or variance is the simple random sample mean or variance without replacement. As stated earlier, if an auxiliary variable X closely related to the study variable Y is available and X is easy to obtain then one can use ratio, product and regression estimators to improve the performance of the estimator of the study variable. Estimating the finite population variance has great significance in various fields such as Industry, Agriculture, Medical and Biological sciences. In this paper, we consider the problem of estimation of the population variance and use the auxiliary information to improve the efficiency of the estimator of population variance.

Before discussing further about the traditional product type variance estimator and the proposed modified product type variance estimators, the notations to be used in this paper are described below:

- N – Population size
- n – Sample size
- $\gamma=\frac{(1-f)}{n}$
- Y – Study variable
- X – Auxiliary variable
- $\bar{X},\bar{Y}$ – Population means
- $\bar{x},\bar{y}$ – Sample means
- S_x^2,S_y^2 – Population variances
- s_x^2,s_y^2 – Sample variances
- $\lambda_{rs}=\frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$
- $\mu_{rs}=\frac{1}{N}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^r\left(X_i-\bar{X}\right)^s$
- $\beta_{1(x)}=\frac{\mu_{03}^2}{\mu_{02}^3}$ Skewness of the auxiliary variable
- $\beta_{2(x)}=\frac{\mu_{04}}{\mu_{02}^2}$
- M_d – Median of the auxiliary variable

- Q_1 – First (lower) quartile of the auxiliary variable
- $B(.)$ – Bias of the estimator
- $MSE(.)$ – Mean squared error of the estimator
- $\hat{S}_p^2$ – Traditional product type variance estimator of S_y^2
- $\hat{S}_{pr_j}^2$ – Proposed modified product type variance estimator of S_y^2

Product type variance estimator for the population variance S_y^2 when the population variance S_x^2 of the auxiliary variable X is known together with its bias and mean squared error are given below:

$$\hat{S}_p^2 = s_y^2 \frac{s_x^2}{S_x^2} \qquad (1.1)$$

$$B\left(\hat{S}_p^2\right) = \gamma S_y^2\left[\left(\lambda_{22} - 1\right)\right] \qquad (1.2)$$

$$MSE\left(\hat{S}_p^2\right) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + \left(\beta_{2(x)} - 1\right) + 2\left(\lambda_{22} - 1\right)\right] \qquad (1.3)$$

where $\beta_{2(y)} = \frac{\mu_{40}}{\mu_{20}^2}, \lambda_{22} = \frac{\mu_{22}}{\mu_{20}\mu_{02}}$

The product type variance estimator given in (1.1) is used to improve the precision of the estimate of the population variance when there exists a negative correlation between X and. The problem of constructing efficient estimators for the population variance has been widely discussed by various authors such as Cochran [5], Wolter [28], Murthy [12], Isaki [9], Das and Tripathi [6], Singh and Chaudhary [16], Agarwal and Sithapit [1], Ahmed et al. [2], Al-Jararha and Al-Haj Ebrahem [3], Arcos et al [4], , Garcia and Cebrain [7], Gupta and Shabbir [8], Kadilar and Cingi [10-11], Prasad and Singh[13], Reddy [14], Shabbir and Gupta [15], Singh and Solanki [17], Singh et al.[18], Sisodia and Dwivedi [19], Subramani and Kumarapandiyan [20-23], Tailor and Shrama [24], Upadhyaya and Singh [25-27], and Yadav and Kadilar [29-30].

The materials of the present study are arranged as given below. The proposed estimators using known parameters of the auxiliary variable are presented in section 2 and the bias and mean square error have been derived in section 3 where as the conditions in which the proposed estimators perform better than the traditional product variance estimators are derived in section 4. The performance of the proposed estimators with that of the traditional product estimator is assessed for certain natural populations in section 5 and the conclusion is presented in section 6.

2. PROPOSED ESTIMATORS

In this section we have suggested a class of modified product type variance estimators using the known parameters of the auxiliary variable for estimating the population variance of the study variable X. The proposed class of modified product type variance estimators $\hat{S}_{pr_j}^2, j = 1,2,3,4,5$ or estimating the population variance S_y^2 is given below:

$$\hat{S}_{pr_j}^2 = s_y^2\left[\frac{s_x^2 + \omega_j}{S_x^2 + \omega_j}\right]; j = 1,2,3,4,5$$

$$\omega_1 = M_d, \omega_2 = Q_1, \omega_3 = Q_3, \omega_4 = \beta_{1(X)}, \omega_5 = \beta_{2(X)}$$

3. DERIVATION OF BIAS AND MEAN SQUARED ERROR

We have derived the bias and mean squared error of the proposed estimators $\hat{S}_{pr_j}^2 : j = 1,2,3,4,5$ to first order of approximation with the following notations:

Let us define $e_0 = \frac{s_Y^2 - S_Y^2}{S_Y^2}$ *and* $e_1 = \frac{s_X^2 - S_X^2}{S_X^2}$. Further we can write $s_Y^2 = S_Y^2(1+e_0)$ and $s_X^2 = S_X^2(1+e_0)$ and from the definition of e_0 and e_1 we obtain:

$$E[e_0] = E[e_1] = 0$$

$$E\left[e_0^2\right] = \gamma\left(\beta_{2(Y)} - 1\right);\ E\left[e_1^2\right] = \gamma\left(\beta_{2(x)} - 1\right)$$

$$E\left[e_0 e_1\right] = \gamma\left(\lambda_{22} - 1\right) \text{ where } \lambda_{22} = \frac{\mu_{22}}{\mu_{20}\mu_{02}}$$

The proposed estimator $\hat{S}_{pr_j}^2 : j = 1,2,3,4,5$ is given below:

$$\hat{S}_{pr_j}^2 = s_y^2\left[\frac{s_X^2 + \omega_j}{S_X^2 + \omega_j}\right]; j = 1,2,3,4,5$$

where

$$\omega_1 = M_d, \omega_2 = Q_1, \omega_3 = Q_3, \omega_4 = \beta_{1(X)}, \omega_5 = \beta_{2(X)}$$

$$\Rightarrow \hat{S}_{pr_j}^2 = S_Y^2\left(1+e_0\right)\left[\frac{S_X^2 + e_1 S_X^2 + \omega_j}{S_X^2 + \omega_j}\right]$$

$$\Rightarrow \hat{S}_{pr_j}^2 = S_Y^2\left(1+e_0\right)\left[\frac{\left(S_X^2 + \omega_j\right)\left(1 + \frac{e_1 S_X^2}{S_X^2 + \omega_j}\right)}{\left(S_X^2 + \omega_j\right)}\right]$$

$$\Rightarrow \hat{S}^2_{pr_j} = S^2_Y(1+e_0)\left(1+A_{p_j}e_1\right) \text{ where } A_{p_j} = \frac{S^2_X}{S^2_X+\omega_j}$$

$$\Rightarrow \hat{S}^2_{pr_j} = \left(S^2_Y + S^2_Y e_0\right)\left(1+A_{p_j}e_1\right)$$

$$\Rightarrow \hat{S}^2_{pr_j} = S^2_Y + S^2_Y e_0 + S^2_Y A_{p_j} e_1 + S^2_Y A_{p_j} e_0 e_1$$

$$\Rightarrow \hat{S}^2_{pr_j} - S^2_Y = S^2_Y e_0 + S^2_Y A_{p_j} e_1 + S^2_Y A_{p_j} e_0 e_1 \quad (3.1)$$

Taking expectation on both sides, we get

$$E\left(\hat{S}^2_{pr_j} - S^2_Y\right) = S^2_Y E(e_0) + S^2_Y A_{p_j} E(e_1) + S^2_Y A_{p_j} E(e_0 e_1)$$

$$\text{Bias}\left(\hat{S}^2_{pr_j}\right) = \gamma\, S^2_Y A_{p_j}(\lambda_{22}-1) \text{ where } A_{p_j} = \frac{S^2_X}{S^2_X+\omega_j}$$

Squaring both the sides of (3.1), neglecting the terms higher than second order and taking expectation on both the sides, we get

$$E\left(\hat{S}^2_{pr_j} - S^2_Y\right)^2 = S^4_Y E\left(e_0^2\right) + S^4_Y A^2_{p_j} E\left(e_1^2\right)$$
$$+2S^4_Y A_{p_j} E(e_0 e_1)$$

$$\text{MSE}\left(\hat{S}^2_{pr_j}\right) = \frac{(1-f)}{n}\begin{pmatrix} S^4_Y\left(\beta_{2(Y)}-1\right) + S^4_Y A^2_{p_j}\left(\beta_{2(X)}-1\right) \\ +2S^2_Y A_{p_j}(\lambda_{22}-1) \end{pmatrix}$$

$$\text{MSE}\left(\hat{S}^2_{pr_j}\right) = \gamma\, S^4_Y \begin{pmatrix} \left(\beta_{2(Y)}-1\right) + A^2_{p_j}\left(\beta_{2(X)}-1\right) \\ +2A_{p_j}(\lambda_{22}-1) \end{pmatrix}$$

The bias and mean squared error of the proposed estimators $\hat{S}^2_{pr_j}$, $j=1,2,3,4,5$ have been derived (see Appendix A) and are given below

$$B\left(\hat{S}^2_{pr_j}\right) = \gamma S^2_y A_{p_j}\left[(\lambda_{22}-1)\right] \quad (3.2)$$

$$\text{MSE}\left(\hat{S}^2_{pr_j}\right) = \gamma S^4_y \begin{bmatrix} \left(\beta_{2(y)}-1\right) + A^2_{p_j}\left(\beta_{2(x)}-1\right) \\ +2A_{p_j}(\lambda_{22}-1) \end{bmatrix} \quad (3.3)$$

where $A_{p_j} = \dfrac{S^2_x}{S^2_x+\omega_j}$

4. EFFICIENCY COMPARISON

The proposed estimators $\hat{S}^2_{pr_j} : j=1,2,3,4,5$ are compared with that of the usual product type variance estimator $\hat{S}^2_p$.

For $\text{MSE}\left(\hat{S}^2_{pr_j}\right) \le \text{MSE}\left(\hat{S}^2_p\right)$

$$\delta S^4_Y\left[\left(\beta_{2(Y)}-1\right) + A^2_{p_j}\left(\beta_{2(X)}-1\right) + 2A_{p_j}(\lambda_{22}-1)\right] \le$$
$$\delta S^4_Y\left[\left(\beta_{2(Y)}-1\right) + \left(\beta_{2(X)}-1\right) + 2(\lambda_{22}-1)\right]$$

$$\Rightarrow \left(\beta_{2(Y)}-1\right) + A^2_{p_j}\left(\beta_{2(X)}-1\right) + 2A_{p_j}(\lambda_{22}-1) \le$$
$$\left(\beta_{2(Y)}-1\right) + \left(\beta_{2(X)}-1\right) + 2(\lambda_{22}-1)$$

$$\Rightarrow A^2_{p_j}\left(\beta_{2(X)}-1\right) + 2A_{p_j}(\lambda_{22}-1) \le$$
$$\left(\beta_{2(X)}-1\right) + 2(\lambda_{22}-1)$$

$$\Rightarrow \begin{Bmatrix} A^2_{p_j}\left(\beta_{2(X)}-1\right) + 2A_{p_j}(\lambda_{22}-1) \\ -\left(\beta_{2(X)}-1\right) - 2(\lambda_{22}-1) \le 0 \end{Bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left(A_{p_j}+1\right)\left(A_{p_j}-1\right)\left(\beta_{2(X)}-1\right) \\ +2(\lambda_{22}-1)\left(A_{p_j}-1\right) \le 0 \end{bmatrix}$$

$$\Rightarrow \left(A_{p_j}-1\right)\left[\left(A_{p_j}+1\right)\left(\beta_{2(X)}-1\right) + 2(\lambda_{22}-1)\right] \le 0$$

Condition 1: $\left(A_{p_j}-1\right) \le 0$ and

$$\left(A_{p_j}+1\right)\left(\beta_{2(X)}-1\right) + 2(\lambda_{22}-1) \ge 0$$

$$A_{p_j}-1 \le 0 \text{ and } \left(A_{p_j}+1\right)\left(\beta_{2(X)}-1\right) \ge -2(\lambda_{22}-1)$$

$$\Rightarrow A_{p_j} \le 1 \text{ and } \left(A_{p_j}+1\right) \ge -\frac{2(\lambda_{22}-1)}{\left(\beta_{2(X)}-1\right)}$$

$$\Rightarrow A_{p_j} \le 1 \text{ and } A_{p_j} \ge -\frac{2(\lambda_{22}-1)}{\left(\beta_{2(X)}-1\right)} - 1$$

$$\Rightarrow -\frac{2(\lambda_{22}-1)}{\left(\beta_{2(X)}-1\right)} - 1 \le A_{p_j} \le 1$$

Condition 2: $\left(A_{p_j}-1\right) \ge 0$ and

$$\left(A_{p_j}+1\right)\left(\beta_{2(X)}-1\right) + 2(\lambda_{22}-1) \le 0$$

$$A_{p_j}-1 \ge 0 \text{ and } \left(A_{p_j}+1\right)\left(\beta_{2(X)}-1\right) \le -2(\lambda_{22}-1)$$

$$\Rightarrow A_{p_j} \ge 1 \text{ and } \left(A_{p_j}+1\right) \le -\frac{2(\lambda_{22}-1)}{\left(\beta_{2(X)}-1\right)}$$

$$\Rightarrow A_{p_j} \geq 1 \text{ and } A_{p_j} \leq -\frac{2(\lambda_{22}-1)}{(\beta_{2(X)}-1)}-1$$

$$\Rightarrow 1 \leq A_{p_j} \leq -\frac{2(\lambda_{22}-1)}{(\beta_{2(x)}-1)}-1$$

That is, MSE $\left(\hat{S}^2_{pr_j}\right) \leq$ MSE $\left(\hat{S}^2_p\right)$, if

$$-\frac{2(\lambda_{22}-1)}{(\beta_{2(X)}-1)}-1 \leq A_{p_j} \leq 1 \quad \text{(or)}$$

$$1 \leq A_{p_j} \leq -\frac{2(\lambda_{22}-1)}{(\beta_{2(x)}-1)}-1$$

5. NUMERICAL STUDY

The performance of the proposed modified product type variance estimators are assessed with that of traditional product type estimator for certain natural populations. The population 1 and 2 are the real data set collected from a Combined Cycle Power Plant for the year 2011. Data and reports are available in the website http://archive.ics.uci.edu/ml/datasets.html [31]. In the data set, three variables are considered: Net hourly electrical energy output (Y) and Temperature (X_1) and Exhaust Vacuum (X_2). The population 3 is a real data of Concrete Compressive Strength consisting of 2 variables: Concrete compressive strength (MPa, megapascals) (Y) and Water (component 4) (kg in a m^3 mixture) (X). Data and reports are available in the website http://archive.ics.uci.edu/ml/datasets.html [31]. The population parameters and the constants computed from the above populations are given below:

Table 1: Parameters and Constants of the Populations

Parameters	Population 1	Population 2	Population 3
N	9568	9568	1030
n	2500	2500	333
$\bar{Y}$	454.3650	454.3650	35.818
$\bar{X}$	19.6512	54.3058	181.5663
S_y	17.0669	17.0669	16.6976
S_x	7.4524	12.7079	21.3452
$\beta_{1(x)}$	0.0186	0.0394	0.0055
$\beta_{2(x)}$	1.9624	1.5557	3.1162
$\beta_{2(y)}$	1.9514	1.9514	2.6818
λ_{22}	1.7665	1.3139	1.0308
A_{p1}	0.7318	0.7561	0.7112
A_{p2}	0.8043	0.7946	0.7342
A_{p3}	0.6834	0.7082	0.7035
A_{p4}	0.9996	0.9997	0.9999
A_{p5}	0.9658	0.9904	0.9932

The biases and mean squared errors of the traditional product type estimator and proposed modified product type variance estimators for the populations given above are given in the following Tables

Table 2: Biases of the existing and proposed modified ratio type variance estimators

Estimator	Bias		
	Population 1	Population 2	Population 3
$\hat{S}^2_p$	0.0659	0.0270	0.0175
$\hat{S}^2_{pr_1}$	0.0482	0.0204	0.0124
$\hat{S}^2_{pr_2}$	0.0530	0.0215	0.0128
$\hat{S}^2_{pr_3}$	0.0450	0.0191	0.0123
$\hat{S}^2_{pr_4}$	0.0659	0.0270	0.0175
$\hat{S}^2_{pr_5}$	0.0637	0.0268	0.0174

Table 3: MSE(.) of the existing and proposed modified ratio type variance estimators

Estimator	Mean Squared Error MSE(.)		
	Population 1	Population 2	Population 3
$\hat{S}^2_p$	86.4131	53.5303	611.1951
$\hat{S}^2_{pr_1}$	64.9057	43.7238	442.8708
$\hat{S}^2_{pr_2}$	70.3755	45.1608	454.2532
$\hat{S}^2_{pr_3}$	61.3918	41.9905	439.1516
$\hat{S}^2_{pr_4}$	86.3841	53.5196	611.1869
$\hat{S}^2_{pr_5}$	83.4827	53.1154	606.5958

From the values of Table 2, it is observed that the biases of the proposed modified product type variance estimators are less than the bias of the traditional product type variance estimator. Similarly from the values of Table 3, it is observed that the mean squared errors of the proposed modified product type variance estimators are less than the mean squared errors of the traditional product type variance estimators.

6. CONCLUSIONS

In this paper a class of modified product type variance estimators using the parameters of the auxiliary variable are known have been proposed. The biases and mean squared errors of the proposed modified product type variance estimators are obtained. Further we have derived the conditions for which the proposed estimators are more efficient than the traditional product type variance estimator. We have also assessed the performances of the proposed estimators for some known natural populations. It is observed from the numerical comparison that the biases and mean squared errors of the proposed estimators are less than the biases and mean squared errors of the traditional product type variance estimator. Hence we strongly recommend that the proposed modified product type variance estimator may be preferred over the traditional product type variance estimator for the use of practical applications.

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